Mr. Michael T. Davis WLPCS Calculus	Section 4.3 Quiz February 22, 2018

Name: Solutions/Answers

1. (4 pts) Find $\frac{dy}{dx}$ given $y=\sin^{-1}(x)$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

2. (4 pts) Find $\frac{dy}{dx}$ given $y=\cos^{-1}(x)$

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$$

3. (4 pts) Find $\frac{dy}{dx}$ given $y=\tan^{-1}(x)$

$$\frac{dy}{dx}=\frac{1}{1+x^2}$$

4. (4 pts) Find $\frac{dy}{dx}$ given $y=\cot^{-1}(x)$

$$\frac{dy}{dx}=\frac{-1}{1+x^2}$$

5. (4 pts) Find $\frac{dy}{dx}$ given $y=\sec^{-1}(x)$

$$\frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}}$$

6. (4 pts) Find $\frac{dy}{dx}$ given $y=\csc^{-1}(x)$

$$\frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2-1}}$$

7. (6 pts) Find $\frac{dy}{dx}$ given $y=\cos^{-1}\left(x^2\right)$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-(x^2)^2}}\cdot 2x=\frac{2x}{\sqrt{1-x^4}}$$

8. (6 pts) Find y' given $y=\sin^{-1}\left(t^2+t\right)$

$$y'=\frac{1}{\sqrt{1-(t^2+t)^2}}\cdot(2t+1)=\frac{2t+1}{\sqrt{-t^4-2t^3-t^2+1}}$$

9. (6 pts) Find $\frac{dy}{dt}$ given $y=2t\tan^{-1}\left(\frac{1}{t}\right)$

$$\frac{dy}{dt}=2\cdot\tan^{-1}\left(\frac{1}{t}\right)+2t\,\frac{1}{1+\left(\frac{1}{t}\right)^2}\cdot\frac{-1}{t^2}$$

$$=2\cdot\tan^{-1}\left(\frac{1}{t}\right)-\frac{2}{t+\frac{1}{t}}$$

$$=2\cdot\tan^{-1}\left(\frac{1}{t}\right)-\frac{2t}{t^2+1}$$

10. (6 pts) Find $\frac{dy}{dx}$ given $y=\sec^{-1}\left(\pi-x\right)$

$$\frac{dy}{dx}=\frac{1}{\sqrt{(\pi-x)^2-1}}\cdot(-1)=\frac{-1}{\sqrt{\pi^2-2\pi x+x^2-1}}$$

11. (6 pts) Find $\frac{dy}{dx}$ given $y=\cot^{-1}\left(\sqrt{2x}\right)$

$$\frac{dy}{dx} = \frac{-1}{1+(\sqrt{2x})^2}\left(\tfrac{1}{2}(2x)^{-\frac{1}{2}}\cdot 2\right) = \frac{-\frac{1}{\sqrt{2x}}}{1+2x}$$

$$= \frac{-1}{\sqrt{2x}\,(1+2x)}$$

12. (6 pts) A particle moves along the x-axis so that its position in meters at any time $t\geq 0$ is given by $x(t)=\sin^{-1}(2t)$. Find the velocity, $v(t)$, of the particle at $t=\frac{1}{4}$ seconds.

$$V(t) = x'(t) = \frac{2}{\sqrt{1-4t^2}}$$

$$V\left(\tfrac{1}{4}\right) = \frac{2}{\sqrt{1-4\left(\frac{1}{16}\right)}} = \frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}} \text{ m/sec}$$

13. (6 pts) A particle moves along the x-axis so that its position in meters at any time $t\geq 0$ is given by $x(t)=\tan^{-1}\left(\frac{t}{2}\right)$. Find the velocity, $v(t)$, of the particle at $t=1$ seconds.

$$V(t) = x'(t) = \frac{\frac{1}{2}}{1+\left(\frac{t}{2}\right)^2} = \frac{\frac{1}{2}}{1+\frac{t^2}{4}} = \frac{2}{4+t^2}$$